

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010G/H University Mathematics 2014-2015
Assignment 3

- Due date: 12 Mar, 2015 (before 17:00)
- Remember to write down your name and student number
- Please work on ALL questions below.

Questions from Thomas Calculus:

Exercise 3.7: 7, 11

Exercise 3.8: 23, 41

1. Let $f(x) = x^{1/3} - \frac{1}{3}x - \frac{2}{3}$ for $x > 0$. Show that $f(x) \leq 0$ for all $x > 0$.
Hence deduce that, for $x, y > 0$,

$$x^{1/3}y^{2/3} \leq \frac{1}{3}x + \frac{2}{3}y.$$

2. Let $f(x) = \frac{x^3}{x^2 - 4}$, where x is a real number and $x \neq \pm 2$.

- Find $f'(x)$ and $f''(x)$ for $x \neq 1$.
- Find the range of x such that
 - $f'(x) > 0$
 - $f'(x) < 0$
 - $f''(x) > 0$
 - $f''(x) < 0$
- Find the local extrema and saddle points, if any.
- Find the points of inflection, if any.
- Find the asymptotes of the graph of $f(x)$, if any.
- Sketch the graph of $f(x)$.